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ELECTROMAGNETIC ACTUATOR AND STACKED PIEZOELECTRIC SENSORS FOR CONTROLLING VIBRATIONS OF A MOTOR ON A FLEXIBLE STRUCTURE

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This paper presents an actuator with a sensor for controlling vibrations of machines. It consists of a voice coil-type electromagnetic actuator connected to a piezoelectric sensor and a coil spring. The actuator is used for controlling a motor laid on the beam. The sensor detects the disturbance force due to the centrifugal force of the motor, and the force, which is equivalent to the disturbance force, is applied to the motor and the beam by the actuator. This implies that the disturbance force is cancelled. The control technique just mentioned has advantages because it reduces the vibration on the low-frequency region. In addition, the actuator becomes compact, and the control force becomes small. But the control is insufficient for controlling the resonance peak. The PD control is available for reducing the resonance peak, but the displacement and velocity sensors are required. In the present article, a method of vibration control of disturbance cancellation combining the PD control is presented in which both the displacement and velocity sensors are not required. The analytical results for obtaining the control current and the response of both the motor and beam have been obtained. To validate the present actuator–sensor system and the control method, experimental tests are carried out.

1. INTRODUCTION

Motors on beams or plates are often observed in practice. The structures such as beams and plates are flexible in general, and so they vibrate due to the centrifugal forces of the motors. The vibration is transmitted to the other portions such as the gear mechanism, so that the accuracy of machines is spoilt. In addition, the vibration often generates large sounds. In order to suppress vibrations and sounds, the motor is usually rigidly connected to the structure, and the structural vibration is suppressed. One popular method is to use vibration absorbers, and various vibration absorbers and dampers have been presented in a number of reports [1–4].

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Recently, complicated phenomena such as non-linear vibrations have been discussed for vibration absorbers [5, 6]. Vibration absorbers have been used for suppressing vibrations of various structures [7, 8], and machines such as offshore platforms [9], valves [10], multi-stage-pumps [11], and shock absorbers of machines [12]. Recently, active and active/passive vibration absorbers have been discussed [13–15] in which rigidity and damping are varied by using actuators. Tunable absorbers have also been proposed in references [17, 18]. Najet *et al.* [16] gave an interesting tunable absorber for a two-degree-of-freedom system in which the optimal poles were given by the use of time delay feedback. The present author has proposed the multi-absorbers optimized with the neural network for controlling vibrations of flexural structures. In that method, however, the system becomes heavy and a space to attach the absorber is required.

In recent years, small and light mechanical systems are required. In order to decrease the sizes of machines and structures, active vibration control is needed. Spencer *et al.*, Van and Sas, Pan and Hansen, Brennan *et al.*, Setora, Liu and Yang have reported various active vibration control methods [20–26]. The sensors and actuators are of importance in active vibration control. Oshima *et al.* [27] presented self-sensing control. Starchville and Thomson [28] discussed the distributed sensor system. Some interesting actuators were also reported. Lee and Jee [29] gave the control using the hybrid damper, and Cheong and Choi [30] presented the shape memory actuator. Murozono and Sumi [31] also discussed the use of thermal bending. One popular actuator is the piezoelectric actuator. Popp and Frischgesell [32], Young and Hansen [33], Fukuda *et al.* [34] and Kim *et al.* gave the methods of vibration control by using piezoelectric actuators. Clark *et al.* [35] also made comparisons of actuators [36]. The problems treated in the papers are concerned with the active vibration control of beams, and hence, large energies, large power amplifiers and strong actuators are required when heavy machines such as a motor lie on the beam. The energy to control vibration is also required to be small.

When the motor is connected to the beam by a soft spring, the transmissibility is reduced, so that the vibration of the beam decreases, while the motor vibration increases. In the system, when the cancellation force corresponding to the centrifugal force acts on the motor, both beam vibrations and motor vibration can be suppressed. But in this system, the control energy is small in comparison with the active control methods as mentioned above. However, the method using the principle has not been used in previous works.

A hybrid control is desirable, in which the vibration at the resonance is controlled, but no control is performed after the resonance. This paper proposes a compact sensor actuator system for controlling vibrations of a motor on a structure based on the principles mentioned above. In order to suppress vibrations of a structure with a motor, small stiffness springs are inserted between the motor and the structure. But vibrations of the motor are large in the system. The damper is available to control both motor and structural vibrations. In the system, however, the transmissibility increases. Especially for the spring–damper system, the transmissibility in the low-frequency region is always greater than one. In order to overcome these shortcomings, this paper discusses a method of vibration control for a structure with a motor. In our previous report, we presented a vibration control method in which the usual PD control is combined with the disturbance cancellation [5]. The method has advantages, because the PD control is combined without consideration of interference between the two controls. The transmissibility can be controlled to be less than one when using the method. In the present system, however, the method cannot be used directly because the structure is a continuous mass system. In this paper, the disturbance cancellation combining the PD control is presented which is applicable to the continuous mass system. When the method is used, two sensors, which detect both displacement and velocity of the motor, but the control system, become complex and expensive. Hence, the sensorless system is presented in which the displacement and the velocity are calculated theoretically. Analytical results are obtained, and numerical calculations are carried out. To validate the system and the control method, experimental tests are also performed.

2. GEOMETRY OF THE SENSOR–ACTUATOR SYSTEM

Figure 1 depicts the system treated in this paper. The main mass (the motor) is on the flexible structure (the beam). The object of this article is to control vibrations of both the motor and the beam. In order to control vibrations, the actuator with the sensor (sensor–actuator system) is presented. Figure 2 shows the sensor–actuator system. The sensor consists of the piezoelectric ceramics, which is stacked inside frame 1 with appropriate compression. The centrifugal force of the motor compresses the piezoelectric ceramics, and then electric voltage is generated during the motor rotation. In this sensor, the upper ceramic detects the upward force and the lower detects the downward force.

The actuator consists of a moving coil-type linear motor as shown in Figure 2. In the actuator, the left side of a rectangular coil is inserted between two plate-type permanent magnets. The N-pole of one magnet faces the S-poles of the other. The right side is the same as the left, but the arrangement of the poles is opposite. Hence,

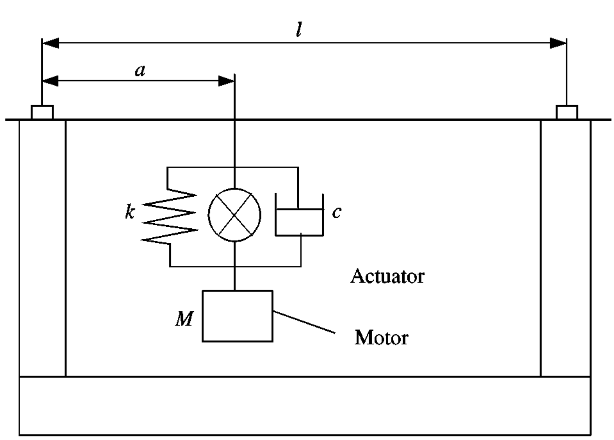

Figure 1. Analytical model.

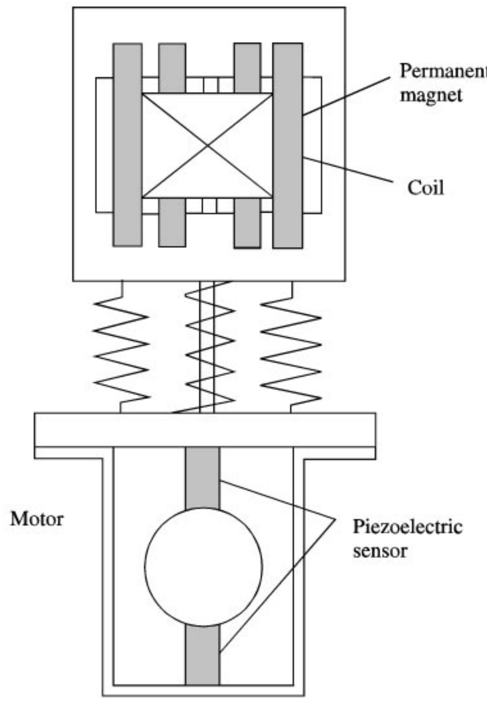

Figure 2. Geometry of the sensor and actuator.

the direction of electromagnetic force of the left side is the same as that of the right side. The magnets are attached to frame 2. A rigid copper bar is connected to the frame of the coil, and the other end of the bar is connected to frame 1. Frame 1 is connected to frame 2 by springs as shown in Figure 2. The top of frame 2 is connected to the beam as shown in Figure 1. Then if an electric current acts on the coil, the coil slides between permanent magnets, and the gap between frames 1 and 2 varies. This implies that the electric current in the coil can control the movement of the motor and the structure in the upward or downward direction.

The sensor and the actuator system are stacked between the motor and the beam, so that the system is compact. Especially, the piezoelectric ceramic is stacked in the rigid frame 1. Then, the centrifugal force of the motor is detected accurately, because the vibrations of the structures are not involved in the sensor. These are the advantages of this sensor–actuator system.

3. DESIGN OF THE CONTROL SYSTEM

Usually, the PD control is used to control vibrations like this system, but it is difficult to suppress vibrations in the low-frequency region. In addition, it requires large damping for passing through the resonance zone. Hence it requires a powerful actuator and the energy loss is large. In order to remove these disadvantages, this paper presents the disturbance cancellation method. In the method, the transfer function for reducing the vibrations of the motor to zero is obtained. The control force of the actuator is created by use of the transfer function. One advantage of this method is that the vibration in the low-frequency region involving the first resonance zone is suppressed remarkably. However, the vibrations at the resonance zone cannot be controlled perfectly by use of this method. Hence, the PD control is also combined with the method mentioned above.

3.1. DISTURBANCE CANCELLATION CONTROL

The vibration can be suppressed perfectly when the force generated by the actuator is the same as that of the negative value of the centrifugal force $Q(t)$ of the motor. The piezoelectric sensor detects the centrifugal force $Q(t)$ in the sensor–actuator system mentioned above. Hence, the force of the moving coil actuator (see Figure 2) under the PD control is

$$u(t) = Ki(t) = Q(t) + f_1 y + f_2 \dot{y}, \tag{1}$$

where $i(t)$ is the control current, K is the coefficient, $u(t)$ the control force, f_1 the displacement feedback coefficient, and f_2 the velocity feedback coefficient. If the beam is rigid, the disturbance force can be cancelled by adding the current $i(t)$ to the actuator. In the present system, however, the beam is flexible, so that the main mass (motor) moves under the control force $u(t)$ due to the deflection of the beam. Hence, the disturbance cannot be cancelled by the method mentioned above. The control force $Q(t)$ to cancel the disturbance has to be obtained by the equations of motion of both the beam and the motor in this system. The equation of motion of the motor is

$$M\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + c\{\dot{y} - \dot{w}(a)\} + k\{y - w(a)\} = -Q(t) + f(t), \tag{2}$$

where $f(t)$ is the centrifugal force, $Q(t)$ the force for cancelling the disturbance, M the mass of the motor, y the motor displacement, w the beam displacement, k the spring constant between the actuator (frame 2) and the sensor (frame 1), c the damping coefficient, and a the length at the motor measured from the left end of the beam. The equation of motion of the beam is

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = R\delta(x - a), \tag{3}$$

where $\delta(x - a)$ is the Dirac delta function, R is the reaction force between the actuator and the beam, EI is the flexural rigidity, ρ is the density and A is the cross-sectional area. Performing the Laplace transformation when all initial conditions are zero, one obtains the displacement $\bar{y}$ in the image domain

$$\bar{y} = \frac{\bar{w}(a,t)(cs + k) - \bar{Q} + \bar{f}}{Ms^2 + cs + k}. \tag{4}$$

if the disturbance force is cancelled perfectly, the displacement y is controlled to be zero:

$$(y) = 0. \tag{5}$$

The force $\bar{Q}$, which cancels the disturbance, is obtained from equations (4) and (5) as

$$\bar{Q} = \bar{w}(a, t)(cs + k) + \bar{f} = \sum_{n=1}^{\infty} \bar{B}_n X_n(a)(cs + k) + \bar{f}. \tag{6}$$

The displacement of the beam is shown to be

$$w(x, t) = \sum_{n=1}^{\infty} B_n(t) X_n(x). \tag{7}$$

When the beam is built-in at both edges, the eigenfunction for the beam is

$$X_n = d_n \{\cosh \xi_n x - \cos \xi_n x - \alpha_n (\sinh \xi_n x - \sin \xi_n x)\}, \tag{8}$$

where

$$\alpha_n = \frac{\sinh \xi_n l + \sin \xi_n l}{\cosh \xi_n l - \cos \xi_n l}, \qquad d_n = \frac{1}{\sqrt{2}} \left(\frac{-\sinh \xi_n l \sin \xi_n l}{\cosh \xi_n l - \cos \xi_n l} \right),$$

$$\xi_1 l = 4{\cdot}73004, \qquad \xi_2 l = 7{\cdot}85321, \qquad \xi_3 l = 10{\cdot}9956, \qquad \xi_4 l = 14{\cdot}1372.$$

Substituting equation (8) into equations (7) and (3), and using the orthogonal characteristic, one obtains the following relation:

$$\bar{B}_n = \frac{2 \bar{R} l^3 X_n(a)}{\{(\xi_n l)^4 - (\lambda l)^4\} EI}, \tag{9}$$

where

$$\lambda^4 = \frac{-\rho A s^2}{EI}.$$

The reaction force R is

$$R = c\{\dot{y} - \dot{w}(a, t)\} + k\{y - w(a, t)\} + Q(t). \tag{10}$$

When the disturbance cancellation is performed, the displacement y and the velocity $\dot{y}$ are zero in theory. Hence, equation (9) becomes

$$\bar{B}_n = \frac{2 \bar{f} l^3 X_n(a)}{EI \{(\xi_n l)^4 - (\lambda l)^4\}}. \tag{11}$$

Substituting equation (11) into equation (6), one obtains the transfer function, which cancels the disturbance

$$G(s) = \bar{Q}/\bar{f} = \frac{(cs + k)l^3}{EI} \sum_{n=1}^{\infty} \frac{2X_n^2(a)}{\{(\xi_n l)^2 + (\lambda l)^2\}\{(\xi_n l)^2 - (\lambda l)^2\}} + 1. \qquad (12)$$

Hence, the control current $I_e(s)$ for cancelling the disturbance is

$$I_e(s) = \frac{1}{K}\left(\frac{cs + k}{EI}\right) l^3 \left[\sum_{n=1}^{\infty} \frac{2X_n^2(a)}{\{(\xi_n l)^2 + (\lambda l)^2\}\{(\xi_n l)^2 - (\lambda l)^2\}} + 1\right] \bar{f}(s), \qquad (13)$$

where $f(t)$ is the force detected by the piezoelectric sensor. Applying Laplace inversion transform on equation (14), one obtains the current

$$i_e(t) = \frac{1}{K}\left[2\int_0^t \sum_{n=1}^{\infty} X_n^2(a)\{\eta_n \cos\delta_n(t-\tau) + k_n j \sin\delta_n(t-\tau)\} f(\tau)\,\mathrm{d}\tau + f(t)\right], \qquad (14)$$

where

$$\eta_n = \frac{c}{\rho A l}, \qquad \kappa_n = \left(\frac{EI}{\rho A l^4}\right)^{1/2} \frac{k}{(\xi_n l)^2} \frac{l^3}{EI},$$

$$\delta_n = \left(\frac{EI}{\rho A l^4}\right)^{1/2} (\xi_n l)^2.$$

3.2. RESPONSE OF THE BEAM UNDER THE COMBINATION CONTROL OF THE DISTURBANCE CANCELLATION AND THE PD CONTROL

The electric current shown in equation (14) cancels the disturbances perfectly in theory. In actual systems, however, there is some control error, so that vibrations cannot be cancelled. In particular, amplitude at resonance is large even the disturbance force is small. In order to suppress vibrations, which remain after the disturbance cancellation, the PD control is combined. The PD control requires the displacement sensor and the velocity sensor. But the economical cost increases significantly when using them. In this article, the displacement and the velocity of the motor are estimated theoretically, so that the sensors are not needed in our system. Let the control error of disturbance force be $\Delta f(t)$. The value is estimated by the experiment. In equation (14) for the disturbance cancellation, there is no term concerning the displacement or the velocity of the vibrating body. This implies that the disturbance force behaves like the external force, and there is no interference between the disturbance cancellation control and the PD control. This is an advantage of this method. The equation of motion of the motor under the PD control excited by the disturbance force $\Delta f(t)$ is

$$M\ddot{y} + c\{\dot{y} - \dot{w}(a)\} + k\{y - w(a)\} = -f_1 y - f_2 \dot{y} + \Delta f(t). \tag{15}$$

The equation of motion for the beam is

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} = R\delta(x - a). \tag{16}$$

Substituting equation (7) into equation (16), and solving the simultaneous differential equations (equations (15) and (16)), one obtains the displacement of the motor:

$$\bar{y} = \frac{\{\sum_{n=1}^{\infty} B_n^* X_n(a)(cs + k) + 1\}\,\Delta\bar{f}}{\{Ms^2 + (k + f_1) + s(c + f_2)\}}, \tag{17}$$

where

$$B_n^* = \bar{B}_n/\Delta\bar{f} \tag{18}$$

and where the coefficient B_n^* are obtained by solving the following expression:

$$\begin{bmatrix} a_{11}+1 & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22}+1 & \cdots & a_{2N} \\ \vdots & & & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN}+1 \end{bmatrix} \begin{Bmatrix} B_1^* \\ B_2^* \\ \vdots \\ B_N^* \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{Bmatrix}, \tag{19}$$

where

$$a_{mn} = \left[1 - \frac{\{(c + f_2)s + (k + f_1)\}}{\{Ms^2 + (k + f_1) + \mathrm{s}(c + f_2)\}}\right]\left[\frac{2l^3 X_m(a)}{\{(\xi_m l)^4 - (\lambda l)^4\}\,EI}\right](cs + k)\,X_n(a),$$

$$b_m = \left[\frac{(c + f_2)s + (k + f_1)}{Ms^2 + (k + f_1) + s(c + f_2)}\right]\left[\frac{2l^3 X_m(a)}{[(\xi_m l)^4 - (\lambda l)^4]\,EI}\right],$$

$$s_{mn}\begin{cases} 0, & n \neq m, \\ 1, & n = m. \end{cases} \tag{20}$$

The displacement with respect to the time is obtained by performing the Laplace inversion transformation.

Equation (20) is applicable to any existing force. In this system, the existing force is the centrifugal force due to the rotation of the motor. Hence, consider the centrifugal force $e(=m_d r)\omega^2$, where m_d is the mass of the rotor, r is the eccentricity of the rotor, and ω is the rotating angular frequency. Then the disturbance force is given by

$$f(t) = e\omega^2 \sin \omega t. \tag{21}$$

When the disturbance cancellation control cancels $(1-g)$ of the disturbance force, the force remaining after the disturbance cancellation is

$$\Delta f(t) = g f(t). \tag{22}$$

Hence, the response of the motor and the beam is

$$y = Y \sin(\omega t + \phi_2), \qquad w = W \sin(\omega t + \phi_1), \tag{23}$$

where

$$Y = \sqrt{Y_R^2 + Y_I^2}, \quad W = \sqrt{W_R^2 + W_I^2}, \quad \tan\phi_1 = Y_I/Y_R, \quad \tan\phi_2 = W_I/W_R$$

and where Y_R denotes the real part and Y_I the imaginary part of the transfer function $G_1(\mathrm{j}\omega)$, and W_R and W_I the imaginary parts, of the function $G_2(\mathrm{j}\omega)$, mentioned below:

$$G_1(\mathrm{j}\omega) = \frac{\{\sum_{n=1}^{\infty} B_n^* X_n(a)(c\mathrm{j}\omega + k) + 1\} g e \omega^2}{\{-M\omega^2 + (k+f_1) + \mathrm{j}\omega(c+f_2)\}},$$

$$G_2(\mathrm{j}\omega) = \sum_{n=1}^{\infty} B_n^* X_n(x) g e \omega^2. \tag{24}$$

B_n^* has the complex value calculated from equation (19). Equation (20) yields

$$a_{mn} = \left[1 - \frac{\{(c+f_2)\mathrm{j}\omega + (k+f_1)\}}{\{M\omega^2 + (k+f_1) + \mathrm{j}\omega(c+f_2)\}}\right] \times \left[\frac{2l^3 X_m(a)}{\{(\xi_m l)^4 - (\lambda l)^4\} EI}\right](c\mathrm{j}\omega + k) X_n(a),$$

$$b_m = \left[\frac{(c+f_2)\mathrm{j}\omega + (k+f_1)}{\{-M\omega^2 + (k+f_1) + \mathrm{j}\omega(c+f_2)\}}\right]\left[\frac{2l^3 X_m(a)}{\{(\xi_m l)^4 - (\lambda l)^4\} EI}\right],$$

$$(\lambda l)^4 = \rho A \omega^2 l^4 / EI.$$

4. CONTROL CURRENTS

4.1. CALCULATION OF DISPLACEMENTS AND VELOCITIES OF THE MAIN MASS

In this system, to simplify the system, the displacement sensor and the velocity sensor are not used. Only the disturbance force is measured by the piezoelectric sensor. Hence, the displacement and the velocity of the main mass have to be obtained without using the sensors.

The period T is decided from the wave of the disturbance force, and then the angular frequency $\omega = 2\pi/T$ is decided. Consider times t_p and t_{p-1}. The centrifugal force is $e\omega^2 = f(t_{p-1})/\sin\omega t_{p-1}$. Substituting this relation into equation (23), one obtains the displacement of the motor

$$y = g\frac{f(t_{p-1})}{\sin\omega t_{p-1}}\sqrt{Y_R^2 + Y_I^2}\sin(\omega t_p + \phi_1). \tag{25}$$

Hence, the velocity is

$$\dot{y} = g\frac{\omega f(t_{p-1})}{\sin\omega t_{p-1}}\sqrt{Y_R^2 + Y_I^2}\cos(\omega t_p + \phi_1). \tag{26}$$

Equations (25) and (26) indicate that the displacement and the velocity at time t_p can be decided by measured signal of the piezoelectric sensor at t_{p-1}.

4.2. CALCULATION OF THE CONTROL CURRENTS

Substituting $f(t) = e\omega^2$ into Equation (14), and performing the Laplace inversion transformation, one obtains the control current

$$i_e(t) = \frac{2e\omega^2}{K}\sum_{n=1}^{\infty} X_n^2(a)\left[\frac{1}{2}\eta_n\left\{\frac{1}{\omega+\delta_n} + \frac{1}{\omega-\delta_n}\right\}\right.$$
$$\left.\times(\cos\delta_n t - \cos\omega t) + \frac{1}{2}k_n\left\{\frac{\sin\delta_n t + \sin\omega t}{\omega+\delta_n} + \frac{\sin\delta_n t - \sin\omega t}{\omega-\delta_n}\right\}\right] + \frac{1}{K}f(t). \tag{26}$$

The function $f(t_p)$ is estimated by using the signal just before the considered time. Let Δt be the small time, and $\Delta f(t)$ be the small increment of $f(t)$. There is the following relation:

$$f(t_p) = \Delta f + f(t_{p-1}) = 2f(t_{p-1}) - f(t_{p-2}). \tag{27}$$

Hence, the control current which cancels the disturbance force at $t = t_p$ is

$$i_e(t_p) = \frac{1}{K}\left\{\frac{f(t_{p-1})}{\sin \omega t_{p-1}}\right\} \sum_{n=1}^{\infty} X_n^2(a)\left[\eta_n\left\{\frac{1}{\omega+\delta_n}+\frac{1}{\omega-\delta_n}\right\}\right.$$
$$\times(\cos\delta_n t_p - \cos\omega t_p) + k_n\left\{\frac{\sin\delta_n t_p + \sin\omega t_p}{\omega+\delta_n}\right.$$
$$\left.\left.+\frac{\sin\delta_n t_p - \sin\omega t_p}{\omega-\delta_n}\right\}\right] + \frac{1}{K}\{2f(t_{p-1}) - f(t_{p-2})\}. \tag{28}$$

4.3. CONTROL CURRENT UNDER THE DISTURBANCE CANCELLATION AND THE PD CONTROL

The control current under the combination of the disturbance cancellation and the PD control at $t = t_p$ is

$$i(t_p) = i_e(t_p) + \frac{f_1}{K}\, y_{t=t_p} + \frac{f_2}{K}\,\dot{y}_{t=t_p}.$$

Substituting the displacement and the velocity in this equation, one obtains

$$i(t) = i_e(t_p) + \left(\frac{f_1}{K}\right)\frac{g f(t_{p-1})}{\sin\omega t_{p-1}}\sqrt{Y_R^2 + Y_I^2}\,\sin(\omega t_p + \phi_1)$$
$$+\left(\frac{f_2}{K}\right)\frac{g\omega f(t_{p-1})}{\sin\omega t_{p-1}}\sqrt{Y_R^2 + Y_t^2}\,\cos(\omega t_p + \phi_2). \tag{29}$$

When the current given in equation (29) is input to the actuator, the combination control of the disturbance cancellation and the PD control can be performed. The resistance R is constant in the actuator, and so the control voltage V_c is $V_s = Ri(t)$ under the assumption that the inductance is negligible. The voltage is created by the computer, and input to the coil of the electromagnetic actuator. In equation (29), coefficients $X_n(a)$, η_n, δ_n and k_n have constant values and hence the values, can be memorized in the computer. This implies that the calculation time of equation (19) can be reduced. But the calculation time cannot be neglected in the real time control. In this system, the control current at t_p is decided by the signal just before the time. Hence, by calculating equation (28) between t_{p-1} and t_p, the current at time t_p is decided. Then, there is no problem about the time delay.

5. NUMERICAL CALCULATIONS AND EXPERIMENTS

In order to validate this system, numerical calculations and experimental tests have been carried out for a typical problem. Table 1 shows the dimensions of the beam and the actuator used in the experiment.

TABLE 1

Young's modulus of the beam E	$2{\cdot}06 \times 10^1$ N/m^2
Density of the beam ρ	$7{\cdot}85 \times 10^3$ kg/m^3
Thickness of the beam h	3·4 mm
Width of the beam b	40 mm
Length of the beam l	600 mm
Length at the motor from the left end of the beam a	200 mm
Mass of the motor M	0·202 kg
Spring constant of the actuator k	1010 N/m
Damping coefficient c	2·5 N s/m
$e(=m_d r)$	0·000 003 kg m

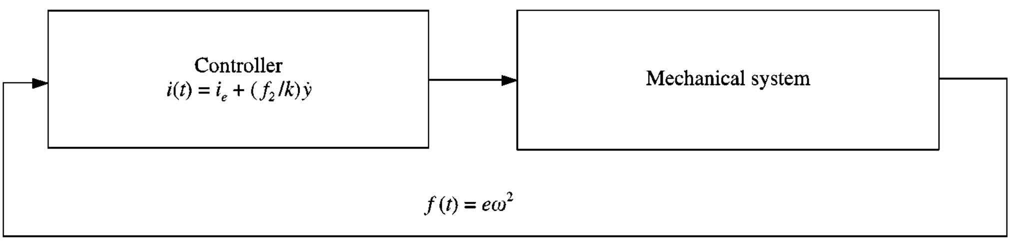

Figure 3. Block diagram of the control system.

The experimental apparatus is the same as in Figure 1. The motor is stacked between two piezoelectric sensors. The actuators is connected to the beam as shown in Figure 1. Figure 3 depicts the block diagram of the control system. In the diagram, the signal (centrifugal force of the motor $f(t) = e\omega^2$) of the piezoelectric sensors is input to the digital signal processor (DSP) with an appropriate sampling period (1 KHz in this experiment). The angular frequency ω is first calculated by using the input vibration wave; then the control current $i(t)$ is calculated using Equation (29) in one sampling period. The voltage corresponding to the current is input to the moving coil actuator. In order to obtain the displacement of the motor under the control, the laser gap sensor is used, and its signal is also input to the oscilliscope, but it is not used in the control of course.

In the experiment, the eccentricity, the mass of the rotor ($e = m_d r$) and the damping coefficient were decided by comparing the experimental response without control and the theoretical response without the control. The experimental response was obtained at the three frequencies lower than the first resonance frequency under the disturbance cancellation only. At the frequencies, the response is stable, and so the effects of disturbance cancellation can be obtained. The response under the disturbance cancellation control was about 40% of the uncontrolled results. Then $g = 0{\cdot}4$ was taken.

This theory involves both the P and D controls, but the effects of the P control on the suppression of the resonance peak is small because the P control has the role of varying the rigidity of the actuator. If appropriate coefficients of both the controls are chosen, the vibrations can be suppressed remarkably. However, the current of

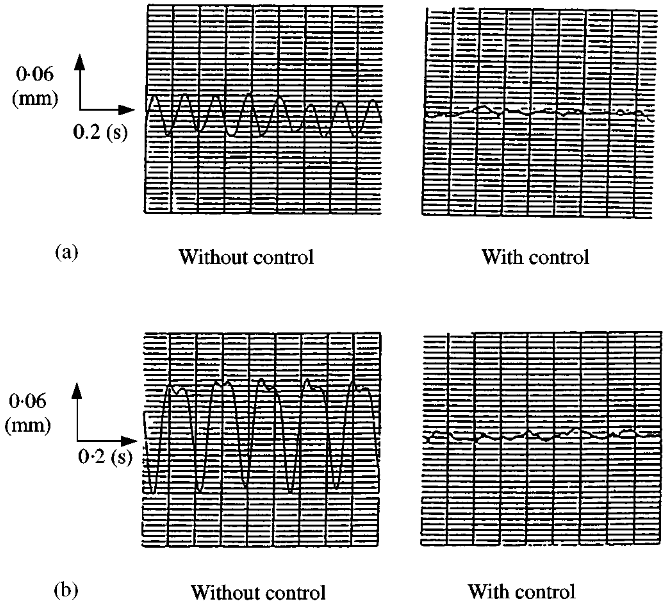

Figure 4. Time response curve of the displacement of the motor. (a) $\omega = 53$ rad/s; (b) $\omega = 71$ rad/s.

the coil of the actuator increases with the coefficients. The actuator used in the experiment was small, and hence the current had to be limited within 5 A. The P and D controls were first investigated, but the effects of the P control were small compared with the D control within the 5A current. Hence, the D control was used in the experiment ($f_1 = 0$). Since in this actuator the piezoelectric ceramics is used as a force sensor, there exists a histolysis. It can be cancelled by using the compressive direction only as shown in Figure 2. There is also a non-linear relationship between the voltage and the force in the piezoelectric ceramics, but it can be assumed to be linear because the displacement of the sensor is small in the experiment.

Figure 4 depicts the time response curves under the combination of our disturbance cancellation and the D control for $f_2 = 6$. The vibration amplitudes are suppressed remarkably in our control as compared to those without control. In the figures, the wave of motor displacement is different from the sine wave due to the higher flexural modes of the beam. In this control, the velocity of the motion of motor is estimated without using the velocity sensor. The velocity is calculated by using the signals of the piezoelectric force sensor. The sensors support the motor in the rigid frame shown in Figure 2. Hence the transmission of the flexural vibration of the beam to the sensor is small. This implies that the correct exciting force of the motor is detected in spite of the beam having higher vibration modes. This is one of the merits of this sensor.

Figure 5 shows the displacement of the motor versus the rotating frequency of the motor for $f_2 = 6$. The solid line illustrates the theoretical results calculated by equation (23), and the white and black dots the experimental results. In this figure, the correct experimental results for the frequencies less than 50 Hz could not be

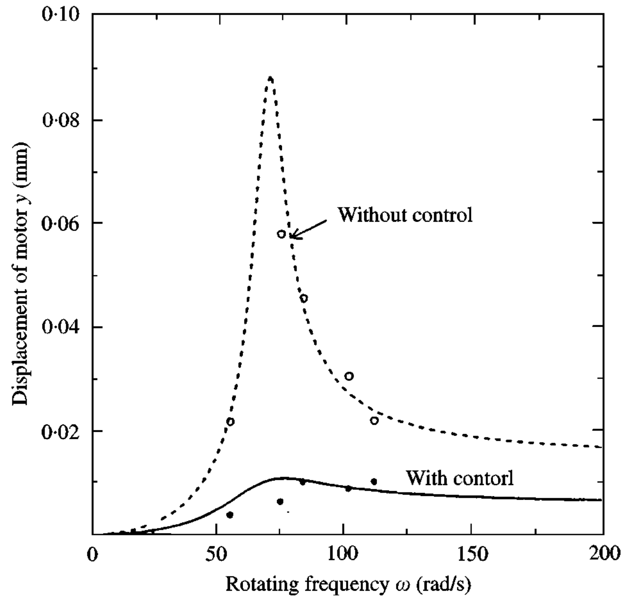

Figure 5. Comparison between theoretical and experimental results for the frequency respone of the displacement of the motor.

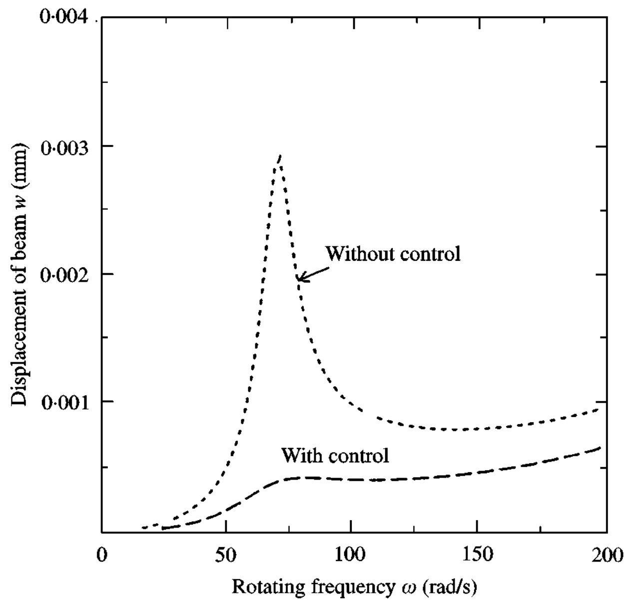

Figure 6. Theoretical frequency response of the displacement at the center of the beam.

obtained because the rotation of the motor was not stable for such small rotation speeds. There is no resonance in the curve of the controlled results. The vibration under the control is suppressed remarkably in comparison with the uncontrolled results to the low-frequency region. However, for the high-frequency range, the actuator did not work well due to the delay of operation. Hence, it is difficult to control vibrations for a high-frequency range. But since the vibration for the frequency greater than the resonance frequency is small, it is not necessary to control it. This implies that the control can be cut after the resonance. In this system, the control has to be made until 130 rad/s, and after that, the control can be cut.

Although a few discrepancies are found between the theoretical and the experimental results, both are in good agreement from a practical engineering point of view. Then, the analytical results can be used in the design of the actuator in practice.

Figure 6 depicts the frequency response of the beam displacement at the center. The vibrations are suppressed remarkably compared to the uncontrolled results. This implies that our control method suppresses the vibrations of both the motor and the beam.

6. CONCLUSION

In this paper, a method for vibration control of the motor on a flexible structure has been presented. The results are summarized as follows:

(1) A linear motor actuator-stacked piezoelectric sensor has been presented.
(2) A method for vibration control of both the motor and the structure has been presented in which the disturbance cancellation and the velocity feedback control are combined.
(3) In the method, the displacement and the velocity sensors are not required. These values are estimated theoretically using the signal of the piezoelectric sensors. Hence, the system is simple and economical as compared to the usual PD control.
(4) The effects of vibration control by our method are significant. The theoretical results are in good agreement with the experimental results and so the control system and the analysis are applicable to the control of vibrations of a structure carrying a motor.

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